№45(б) $\operatorname{rot}\frac{\bar p\times\bar r}{r^3}$

$$\operatorname{rot}(\bar p\times\bar r)=\begin{vmatrix}\frac{\bar e_r}{r^2\sin\nu} & \frac{\bar e_\theta}{r\sin\nu} & \frac{\bar e_\alpha}{r}\\ P_r & rP_\theta & r\sin\nu\cdot P_\alpha\\ \cancel{r} & \cancel{r\theta} & \cancel{r\sin\nu\cdot\alpha}\end{vmatrix}=\frac{\vec e_r}{r^2\sin\theta}(rP_\theta-r\sin\nu\cdot\alpha-$$

$$-r\theta\cdot r\sin\nu\cdot P_\alpha)-\frac{\vec e_\theta}{r\sin\theta}(P_r\cdot r\sin\nu\cdot\alpha-r\cdot r\sin\nu\cdot P_\alpha)+$$

$$+\frac{\vec e_\alpha}{r}(P_r\cdot r\theta-r\cdot rP_\theta)=\vec e_r(P_\theta\alpha-P_\alpha\theta)-\vec e_\theta(P_r\alpha-P_\alpha r)+$$

$$+\vec e_\alpha(P_r\theta-rP_\theta)$$

$$\operatorname{rot}\left(\frac{\bar p\times\bar r}{r^3}\right)=\begin{vmatrix}\frac{\bar e_r}{r^2\sin\nu} & \frac{\bar e_\theta}{r\sin\nu} & \frac{\bar e_\alpha}{r}\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\alpha}\\ \frac{\alpha P_\theta-\theta P_\alpha}{r^2} & \left(\frac{rP_\alpha-\alpha P_r}{r^3}\right)^2 & \frac{\theta P_r-rP_\theta}{r^3}r\sin\nu\end{vmatrix}=$$

$$=\frac{1}{r^3}\left(\frac{\vec e_r}{r^2\sin\theta}\left(\frac{\partial}{\partial\theta}(r\sin\nu\,\theta P_r-2r\sin\nu P_\theta)-\frac{\partial}{\partial\alpha}(2rP_\alpha-r\alpha P_r)\right)\right.-$$

$$-\frac{\vec e_\theta}{r\sin\nu}\left(\frac{\partial}{\partial r}(\theta r\sin\nu P_r-r^2\sin\nu\cdot P_\theta)-\frac{\partial}{\partial\alpha}(\alpha P_\theta-\theta P_\alpha)\right)+$$

$$\left.+\frac{\vec e_\alpha}{r}\left(\frac{\partial}{\partial r}(2rP_\alpha-r\alpha P_r)-\frac{\partial}{\partial\theta}(\alpha P_\theta-\theta P_\alpha)\right)\right)=\frac{1}{r^3}\left(\frac{\bar e_r}{\partial r}(2rP_\alpha-\right.$$

$$\left.-r\alpha P_r-\frac{\bar e_\theta}{r\sin\nu}(-r\sin\nu P_\theta-P_\theta)+\frac{\bar e_\alpha}{r}(rP_\alpha+P_\alpha)\right)=$$

$$=\frac{1}{r^3}\left(\bar e_r\left(\frac{P_r}{r}+\frac{P_r}{r\sin\nu}\right)+\bar e_\theta\left(P_\theta+\frac{P_\theta}{r\sin\nu}\right)+\bar e_\alpha\left(P_\alpha+\frac{P_\alpha}{r}\right)\right)$$

№43 а) $(\bar a\cdot\bar r)\,\bar b$

$\bar a\cdot\bar r = a_x x + a_y y + a_z z$

$(\bar a\cdot\bar r)\bar b = \{(a_1x+a_2y+a_3z)b_1;\ (a_1x+a_2y+a_3z)b_2;\ (a_1x+a_2y+a_3z)b_3\}$

$\operatorname{div}((\bar a\cdot\bar r)\bar b) = a_1b_1+a_2b_2+a_3b_3 = \bar a^3\cdot\bar b^3$

$$\operatorname{rot}((\bar a\cdot\bar r)\bar b) = \begin{vmatrix} \bar i & \bar j & \bar k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (\bar a\cdot\bar r)b_1 & (\bar a\cdot\bar r)b_2 & (\bar a\cdot\bar r)b_3 \end{vmatrix} = \bar i(a_2b_3-a_3b_2)+$$

$$+\bar j(a_3b_1-a_1b_3)+\bar k(a_1b_2-a_2b_1)=\bar a\times\bar b$$

б) $(\bar a\cdot\bar r)\bar r = \{(a_1x+a_2y+a_3z)x;\ (a_1x+a_2y+a_3z)y;\ (a_1x+a_2y+a_3z)z\}$

$\operatorname{div}((\bar a\cdot\bar r)\bar r) = 2a_1x+a_2y+a_3z+2a_2y+a_1x+a_3z+a_1x+$

$+a_2y+2a_3z = 4\bar a\cdot\bar r$

$$\operatorname{rot}((\bar a\cdot\bar r)\bar r) = \begin{vmatrix} \bar i & \bar j & \bar k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (\bar a\cdot\bar r)x & (\bar a\cdot\bar r)y & (\bar a\cdot\bar r)z \end{vmatrix} = \bar i(a_2z-a_3y)+\bar j(a_3x-$$

$$-a_1z)+\bar k(a_1y-a_2x) = \{a_2z-a_3y;\ a_3x-a_1z;$$

$$a_1y-a_2x\} = \cancel{\bar a^3\,\bar r^3}\quad \vec a\times\vec r$$

в) $\operatorname{div}(\bar a\times\bar r)=0$

$$\operatorname{rot}(\bar a\times\bar r) = \begin{vmatrix} \bar i & \bar j & \bar k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (a_2z-a_3y) & (a_3x-a_1z) & (a_1y-a_2x) \end{vmatrix} = \bar i(\bar a_1+\bar a_1)+\bar j(\bar a_2+\bar a_2)+$$

$$+\bar k(\bar a_3+\bar a_3) = 2\vec a$$

г) $\cancel{(\bar a\times\bar r) = \nabla(\varphi(r)\cdot(\vec a\times\vec r)) + \nabla(\varphi(r))\cdot(\bar a\times\bar r) = 0}$